

# Math help desk(4)

## **Department of Electrical Engineering**

# **Contents:**

- Laplace Transform
- Use of Laplace transforms to solve linear, constant coefficient differential equations.
- Inverse Laplace transforms.
- MATLAB applications

- The Laplace transform is a widely used integral transform in mathematics that transforms the mathematical representation of a function in time into a function of complex frequency
- In physics and engineering it is used for analysis of linear timeinvariant [LTI]systems such as electrical circuits and mechanical systems.
- Given a simple mathematical or functional description of an input or output to a system, the Laplace transform provides an alternative functional description that often simplifies the process of analyzing the behavior of the system, or in synthesizing a new system. So, for example, Laplace transformation from the time domain to the frequency domain transforms differential equations into algebraic equations and convolution into multiplication.

#### **Laplace Transform**

Suppose that f(t) is a continuous function. The Laplace transform of f(t) is defined as

$$F(s) = \int_0^\infty e^{-st} f(t) \, dt$$

 $s = \sigma + j \omega$ 

where

Now, the integral in the definition of the transform is called an **improper integral** and it would probably be best to recall how these kinds of integrals work before we actually jump into computing some transforms

#### Remember that you need to convert improper integrals to limits.

Laplace transform of a function f(t) can be obtained with Matlab's function laplace.

Syntax: L =laplace(f)

f(t)	$F(s) = \int_0^\infty f(t) e^{-st} dt$
1	1 5
t	$\frac{1}{s}$ $\frac{1}{s^2}$
t <sup>2</sup>	2 : 5 <sup>3</sup>
t <sup>3</sup>	$\frac{3}{s^4}$
t <sup>n</sup>	n! s <sup>n.1</sup>
U <sub>0</sub> (t)	1 5
U <sub>c</sub> (t)	<u>e-cs</u>
eat	$\frac{e^{-2s}}{s}$ $\frac{1}{s-a}$
te <sup>at</sup>	$\frac{1}{(s-a)^2}$
t <sup>2</sup> e <sup>at</sup>	$\frac{2!}{(s-a)^3}$
t <sup>n</sup> e <sup>at</sup>	$\frac{\mathbf{n}:}{(\mathbf{s}-\mathbf{a})^{\mathbf{n}+1}}$
δ(t)	1
δ(t - a)	e <sup>-as</sup> U <sub>a</sub> (t)
cos (bt)	$\frac{s}{b^2 + s^2}$
sin (bt)	$\frac{\mathbf{b}}{\mathbf{b}^2 + \mathbf{s}^2}$
t <sup>2</sup> cos (bt)	$\frac{2  \mathbf{s}  (\mathbf{s}^2 - 3  \mathbf{b}^2)}{(\mathbf{b}^2 + \mathbf{s}^2)^3}$
t²sin (bt)	$\frac{2 \mathbf{b} (3 \mathbf{s}^2 - \mathbf{b}^2)}{(\mathbf{b}^2 + \mathbf{s}^2)^3}$
e <sup>at</sup> cos (bt)	$\frac{\mathbf{s}-\mathbf{a}}{(\mathbf{s}-\mathbf{a})^2+\mathbf{b}^2}$
e <sup>at</sup> sin (bt)	$\frac{\mathbf{b}}{(\mathbf{s}-\mathbf{a})^2+\mathbf{b}^2}$

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## Find the laplace transform:

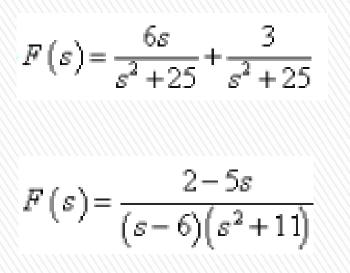
$$f(t) = 6e^{-5t} + e^{3t} + 5t^3 - 9$$

 $g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t)$ 

#### **Inverse laplace transforms**

 $f(t) = \mathcal{L}^{-1}\{F(s)\}$ 

### Find the inverse laplace transform:



# Solution of Differential Equations

But before proceeding into differential equations we will need one more formula. We will need to know how to take the Laplace transform of a derivative.

$$\mathfrak{L}\left\{f^{(n)}\right\} = s^{n}F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \dots - sf^{(n-2)}(0) - f^{(n-1)}(0)$$

$$\mathcal{L}\left\{y'\right\} = sY(s) - y(0)$$
$$\mathcal{L}\left\{y'\right\} = s^2Y(s) - sy(0) - y'(0)$$

#### Solve IVP's with Laplace Transforms

#### Examples:

Solve y' -3y=exp (3x)y(0)=0Solve y''+5y'+6y=0y(0)=2, y'(0)=3Solve y''-10y'+9y=5ty(0)=-1, y'(0)=2Solve y''+3y'+2y=exp(-t)y(0)=4, y'(0)=5

Solve 2y"+3y'-2y=t exp(-2t) y(0)=0,y'(0)=-2

Thank you